**Notesheet - Systems of Equations Circles and Lines**

Definition of a system of equations:

Example of a linear system

\[ y = x + 1 \quad \text{and} \quad y = -3x + 9 \]

Today we will look at systems involving a circle and a line

**Systems of Equations Involving a Circle and a Line**

Solving a system with a circle and a line is very similar to solving a linear system. While a few different methods can be used to solve a linear system, we will be focusing on the **Substitution Method** in this case. Our goal is to identify the ordered pairs which mark any intersection of a given circle and a line. Here are three types of situations.
The Substitution Method for a System Involving a Circle and a Line

**Step 1:** Solve the linear equation for \( y \). Solving for \( y \) is the same as getting the equation into slope-intercept form.  
**Note:** If the equation is of the form, \( x = a \), substitute the value of \( x \) into the circle equation to find your point(s).

**Step 2:** Substitute the value for \( y \) into the circle equation and solve for \( x \). Use either factoring or the quadratic formula to solve. The quadratic formula is:

\[
given \ ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{for all values } a \neq 0.
\]

**Step 3:** Substitute the value(s) for \( x \) back into the linear equation and solve for \( y \).

**Step 4:** Check your solution(s) by substituting your point(s) in both given equations.

**Example 1** Solve the following system of a circle and a line by finding the intersection points algebraically.

\[
\begin{align*}
  y &= 3x - 30 \\
  x^2 + y^2 &= 100
\end{align*}
\]
Practice:

1. \[
\begin{align*}
    y &= 3x - 5 \\
    x^2 + y^2 &= 25
\end{align*}
\]

2. \[
\begin{align*}
    2x + y &= 15 \\
    (x - 2)^2 + (y - 1)^2 &= 25
\end{align*}
\]
3. \[ \begin{align*}
\begin{cases}
  y &= x - 4 \\
  (x + 2)^2 + y^2 &= 4
\end{cases}
\end{align*} \]